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Analysis and prediction of soybean meal price based on uncertain time series

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Abstract: The fluctuation of soybean meal futures price has a profound impact on soybean crushing, poulty and livestock breeding and other related enterprises. Based on the uncertainty theory and time series analysis, an uncertain autoregressive moving average model is established. Based on the data of soybean futures main contract price from 2001 to 2016 in Dalian Commodity Exchange of China, we predict the trend of soybean meal future price by the proposed method. The result shows the reliability of the proposed model.
Keywords: ARMA model, prediction, soybean meal futures price, time series, uncertainty theory.

1. Introduction

Soybean meal is a kind of product for animal feed and vegetable oil meal, which is the largest and most widely used in China. As an important agricultural product, the fluctuation of its price has a great influence on the breeding of main livestock products such as pigs. In order to stabilize the soybean meal price and guide the production of Agricultural Enterprises, Dalian Commodity Exchange listed the soybean meal futures contract in July 2000. This paper attempts to establish an uncertain time series model to analyze the soybean meal futures price in the past years. And then we make the short-term forecast, to verify the reliability of this model.

Time series is a set of observation data arranged in time sequence. There are two basic time series models, one is the autoregressive model proposed by Yule[1], and the other is the moving average model proposed by Walker[2]. Other time series models can be derived from these two basic models, such as the autoregressive moving average model (Whittle[3]), the autoregressive summation moving average model (Box and Jenkins[4]) and the autoregressive conditional heteroskedastic model (Engle[5]). With the development of theory and methods, time series analysis has become not only an important subject in mathematics, but also an important application in many fields such as engineering, sociology, especially economics and finance.

We notice that the traditional time series analysis is aimed at the precisely observed data, but in real life, the data we get is often imprecise, incomplete, or even missing. If we want to process such kind of data, sometimes we have to bring in experts' opinions. Liu[6] proposed the uncertainty theory, and perfected it[7]. In this theory, the experts' opinion can be treated as uncertainty variables to describe the imprecise observations.

Uncertain statistics is an important domain of uncertainty theory. In order to estimate the unknown parameters in an uncertainty model, Liu[8] suggested the principle of the least squares, and Wang and Peng[9] presented the method of moments. Yao and Liu[10] introduced a point estimation of the unknown parameters in uncertain regression models by the principle of the least squares, and Liu and Yang[11] applied the principle of least absolute. After that, Lio and Liu[12] analyzed the residuals and confidence intervals of uncertain regression models. Furthermore, a cross-validation method for calculating the performance of uncertain regression models was proposed by Liu and Jia[13].

Yang and Liu[14] proposed an uncertain autoregressive model to predict future values based on imprecise observations. Lio and Liu[15] proposed the uncertain maximum likelihood estimation to estimate certain parameters in the model. In the financial sector, the prices of financial products are highly uncertain, and these prices can also be seen as a synthesis of the opinions of experts, such as investors, analysts. It is necessary for uncertainty theory to combine with time series analysis, to study the history movements of the price and their impact on the present.

The rest of the paper is organized as follows: Some basic theories and properties is introduced in section 2.The basic model of the time series is proposed and fitted in section 3. In section 4, we estimate the parameters of the model by using the least squares method and maximum likelihood estimation. Finally, we use the model to make predictions of soybean meal price in section 5 and a summary is provided in section 6.

2. Preliminaries

This section introduces some basic theories and properties of uncertainty theory and time series, which will be used later in this article.

Definition 1. (Liu[6]) Let L be a σ -algebra on a nonempty set Γ . A set function $M : L \to [0,1]$ is called an uncertain measure if it satisfies the following three axioms,

Axiom 1: $M\{\Gamma\} = 1$ for the universal set Γ **;**

Axiom 2: $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event Λ **;**

Axiom 3: For every countable sequence of events $\Lambda_1, \Lambda_2, \cdots,$ we have**:**

$$M\left\{\bigcup_{i=1}^{\infty} \Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}$$

In order to provide the operational law, Liu[10] defined the product uncertain measure on the product σ -algebra L **,** called product axiom.

Axiom 4: Let (Γ_k, L_k, M_k) be uncertainty spaces for $k = 1,2,\cdots$. The product uncertain measure M is an uncertain measure satisfying:

$$M\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\} \tag{1}$$

where Λ_k are arbitrarily chosen events from L_k for $k = 1,2,\cdots$, respectively.

Definition 2. (Liu[7])The uncertain variables $\xi_1, \xi_2, \cdots, \xi_m$ are said to be independent if

$$M\left\{\bigcap_{i=1}^{m} \{\xi_i \in B_i\}\right\} = \bigwedge_{i=1}^{\infty} M\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \cdots, B_m$ of real numbers.

Definition 3. (Liu[6])The uncertain distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = M\{\xi \le x\}$$

for any real number x **.**

The uncertain times series is a sequence of uncertain variable. It was proposed by Yang and Liu[14]. Here are some theorems and definitions about uncertain times series following, which will be used for the next part of paper.

Maximum likelihood estimation is a commonly used method to estimate parameters, and likelihood function is used to describe the uncertain measure of occurrence of sample data when uncertain variables are known. Generally speaking, the value of the parameter that maximizes the likelihood function has the greatest measure, and is regarded as the estimation result.

Definition 4. (Lio and Liu[15])Let $\xi_1, \xi_2, \cdots, \xi_n$ be iid samples with uncertainty distribution $F(z \mid \theta)$ where θ is a set of unknown parameters, and let $z_1, z_2, \cdots, z_n$ be observed. Then, the likelihood function is defined by

$$L(\theta \mid z_1, z_2, \cdots, z_n) = \lim_{\Delta z \to 0} \bigwedge_{i=1}^{n} \frac{F(z_i + \frac{\Delta z}{2} \mid \theta) - F(z_i - \frac{\Delta z}{2} \mid \theta)}{\Delta z}$$

Theorem 1. (Lio and Liu[15])Suppose $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $F(z \mid \theta)$ where θ is a set of unknown parameters. Given that $z_1, z_2, \cdots, z_n$ are observed, if $F(z \mid \theta)$ is differentiable at $z_1, z_2, \cdots, z_n$, then the likelihood function is

$$L(\theta \mid z_1, z_2, \cdots, z_n) = \bigwedge_{i=1}^{n} F'(z_i \mid \theta) \tag{2}$$

3. Build model

3.1 Smoothing the sequence

As a bulk agricultural product, soybean meal's price has a certain degree of stability, compared with stocks, options and other financial products prices. Most financial data are highly uncertain, and can be influenced by many events easily. In order to get a more general situation, We should smooth the price sequence $\{Y_t\}$ $(1 \le t \le N)$, by moving average as follows:

$$\begin{cases} Z_1 = Y_1 \\ Z_k = \frac{1}{3}\sum_{j=k-1}^{k+1} Y_j \\ Z_N = Y_N \end{cases} \tag{3}$$

where $2 \le k \le N-1$. And then we get the smoothed timeseries $\{Y_t\}$, $(1 \le t \le N)$.

Definition 5. For an uncertain time series with period T, the season factor S_t of the t-th term $(0 < t \le N)$ is

$$S_t = S_p = \frac{\frac{1}{m}\sum_{j=kT+p}^{N} x_j}{\frac{1}{N}\sum_{i=1}^{N} x_i} \tag{4}$$

where N is the number of samples, $t = kT + p$, k is an integer and $0 \le k < (N/T)$, $p = 1,2,\cdots,T-1$, x_i, x_j are observed values of uncertain time series, m is the number of x_j that meet the criteria.

Because of the influence of economic cycle, climate change, policy change and so on, the price of soybeanmeal has certain periodicity and uncertainty. For these reasons, we add an uncertain term X_t to the modelthat fits the soybean meal price time series:

$$X_t = Z_t - T_t - S_t$$

where Z_t represents the soybean meal futures price, S_t represents the season factors (or cycle factors), T_t represents the trend term, which will be discussed in next subsection.

In other words, The model we supposed to describe the soybean meal price time series is

$$Z_t = T_t + S_t + X_t \tag{5}$$

3.2 Calculating trend term and season factors

We extract the trend term of uncertain time series by polynomial fitting method. In order to fit the trend of uncertain time series better and avoid over fitting, we choose quartic polynomial to fit the uncertain time series. From Equation (5) we get:

$$W_t = Z_t - T_t = X_t + S_t \tag{6}$$

Generally speaking, when we extract the trend term, the sequence will become stable. For the periodicity contained in the time series, we can extract it by the following method. Firstly, we select the period of the time series by Algorithm 1 as follow:

Algorithm 1. Determination of the period for uncertain time series.

Step 1: Let the number of uncertain time series data is N, the mean value and standard deviation of time series are e and σ respectively. Make the threshold value $TV = e + k\sigma$, $k = 1$.

Step 2: Find the items in the time series which is larger than TV, and record their value x_i and the corresponding index i. Put x_i into set A.

Step 3: If the number of elements in set A is more than $0.1*N$ and less than $0.2*N$.

Turn to step 4.

If the number of x_i is more than $0.2*N$, the data may be too discrete and the threshold value may be too small, let $k = k+1$.

If the number of xi is less than $0.1*N$, it means that the degree of dispersion of the data is small, and the deviation of individual data are too large, and the threshold value is too small, then $k = k/2$.

Turn to step 2.

Step 4: Group the elements in set A . $\forall x_i \in A$, $\exists x_j \in A$, s.t. $|i-j|=1$, then x_i and x_j are in the same group, other-wise they are in different groups. Then choose the largest element of each group and put it into the set B .

Step 5: Sort the index of the elements in B , as $n_1, n_2, \cdots, n_k$. By calculating the difference between two adjacent numbers in sequence, we get $m_1 = n_2 - n_1$, $m_2 = n_3 - n_2$, … , $m_{k-1} = n_k - n_{k-1}$ and sort them in array C .

Step 6: Take the median m_p in C and put the elements whose index falls into the interval $[0.1p, 0.9(k-1)+0.1p]$ into the set D .

Step 7: We assume that $D = r_1, r_2, \cdots, r_q$, the period T will be the nearest integer to $\frac{1}{q}\sum_{i=1}^{q} x_i$.

According to Algorithm 1 and Equation (2), we can calculate the season factors S_t . From Equation (6) we get:

$$X_t = W_t - S_t = Z_t - T_t - S_t \tag{7}$$

After extracting the season factors, the time series would have weaker periodicity, and the peaks will be no longer prominent as before.

3.3 Fitting uncertain term

In this paper, the uncertain time series model we used to fit the remaining term X_t is ARMA model:

$$X_t = \sum_{i=1}^{p} a_i X_{t-i} + \varepsilon_t - \sum_{j=1}^{s} b_j \varepsilon_{t-j} \tag{8}$$

where a_i and b_i are model parameters, and ε_i is a disturbance term. If we introduce the delay operators, the Equation (8) can be transformed into:

$$A(q^{-1})X_t = B(q^{-1})\varepsilon_t \tag{9}$$

where

$$A(q^{-1}) = 1 - a_1 q^{-1} - \cdots - a_p q^{-p}$$

is a polynomial of autoregressive coefficient of order p, and

$$B(q^{-1}) = 1 - b_1 q^{-1} - \cdots - b_s q^{-s}$$

is a polynomial of moving average coefficient of order s , and $q^{-k} X_t = X_{t-k}$. If the two polynomials are stable, then the $ARMA(p,s)$ model is also stable and reversible, and then Equation (9) can be transformed into:

$$\varepsilon_t = \frac{A(q^{-1})}{B(q^{-1})} X_t = C(q^{-1}) X_t$$

where $C(q^{-1}) = 1 - \sum_{i=1}^{n} c_i q^{-i}$. Since the characteristic root of the stationary $ARMA(p,s)$ model are all in the unit circle, c_i should decrease exponentially, and $\lim_{i\to\infty}|c_i| = 0$. So if we let $i = n$, which is large enough, the infinite order model can be written as:

$$\begin{aligned}\varepsilon_t &= (1 - c_1 q^{-1} - c_2 q^{-2} - \cdots - c_n q^{-n}) X_t \\ &= X_t - c_1 X_{t-1} - \cdots - c_n X_{t-n}\end{aligned}$$

i.e.

$$X_t = \sum_{i=1}^{n} c_i X_{t-i} + \varepsilon_t \tag{10}$$

4. Parameter estimation

First, we use the least squares to estimate the parameters in the model. The least squares estimate of $c_0, c_1, \cdots, c_k$ in the model (10) is the solution of the minimization problem:

$$\min_{c_i} \sum_{t=n+1}^{N} E\left[\left(X_t - \sum_{i=1}^{n} c_i X_{t-i}\right)^2\right] \tag{11}$$

where N is the number of samples. And then, we can use maximum likelihood method to estimate the mean and variance of the uncertain term.

Theorem 2. Suppose $\xi_1, \xi_2, \cdots, \xi_n$ are iid samples which are normal uncertain variable $N(e, \sigma)$ with unknown parameters e and σ, and $x_1, x_2, \cdots, x_n$ are observed values of them, respectively. Then the maximum likelihood estimators of e and σ satisfy:

$$e = \frac{\bigvee_{i=1}^{n} x_i + \bigwedge_{i=1}^{n} x_i}{2} \tag{12}$$

and σ is the solution of equation:

$$tu - 1 = \frac{2}{e^{tu} - 1} \tag{13}$$

where

$$\begin{cases} t = \min_{e} \bigvee_{i=1}^{n} |e - x_i| \\ u = \dfrac{\pi}{\sqrt{3}\sigma} \end{cases}$$

Proof. Since $\xi_1, \xi_2, \cdots, \xi_n$ are normal uncertain variable $N(e, \sigma)$, their distribution function is

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}$$

According to Theorem 1, we can get the likelihood function of this problem:

$$L(\theta \mid x_1, x_2, \cdots, x_n) = \frac{\dfrac{\pi}{\sqrt{3}\sigma} \exp\left(\dfrac{\pi}{\sqrt{3}\sigma} \bigvee_{i=1}^{n} |e - x_i|\right)}{\left(1 + \exp\left(\dfrac{\pi}{\sqrt{3}\sigma} \bigvee_{i=1}^{n} |e - x_i|\right)\right)^2} \tag{14}$$

So the maximum likelihood estimators e^* and σ^* are the solutions of maximum problem:

$$\max_{e, \sigma > 0} \frac{\dfrac{\pi}{\sqrt{3}\sigma} \exp\left(\dfrac{\pi}{\sqrt{3}\sigma} \bigvee_{i=1}^{n} |e - x_i|\right)}{\left(1 + \exp\left(\dfrac{\pi}{\sqrt{3}\sigma} \bigvee_{i=1}^{n} |e - x_i|\right)\right)^2}$$

Thus, e^* solves problem:

$$\min_{e} \bigvee_{i=1}^{n} |e - x_i| \tag{15}$$

i.e.

$$e^* = \frac{\bigvee_{i=1}^{n} x_i + \bigwedge_{j=1}^{n} x_j}{2} \tag{16}$$

According to Equation (15) and Equation (16), we can get that:

$$\min_{e} \bigvee_{i=1}^{n} |e - x_i| = \frac{\bigvee_{i=1}^{n} x_i - \bigwedge_{j=1}^{n} x_j}{2}$$

And σ^* is the solution of maximum problem:

$$\max_{e,\sigma>0} \frac{\frac{\pi}{\sqrt{3}\sigma}\exp\left(\frac{\pi}{\sqrt{3}\sigma}\bigvee_{i=1}^{n}|e^* - x_i|\right)}{\left(1+\exp\left(\frac{\pi}{\sqrt{3}\sigma}\bigvee_{i=1}^{n}|e^* - x_i|\right)\right)^2}$$

Let $t = \frac{\bigvee_{i=1}^{n} x_i - \bigwedge_{j=1}^{n} x_j}{2}$, and we suppose that:

$$f(\sigma) = \frac{\frac{\pi t}{\sqrt{3}\sigma}\exp\left(\frac{\pi t}{\sqrt{3}\sigma}\right)}{\left(1+\exp\left(\frac{\pi t}{\sqrt{3}\sigma}\right)\right)^2} \tag{17}$$

In order to find the maximum value of $f(\sigma)$, we can get the following equation:

$$\frac{\mathrm{d}f(\sigma)}{\mathrm{d}\sigma} = 0 \tag{18}$$

According to Equation (17) and (18), we can get:

$$\frac{\pi t}{\sqrt{3}\sigma^*}\left(\exp\left(\frac{\pi t}{\sqrt{3}\sigma^*}\right)-1\right) = \exp\left(\frac{\pi t}{\sqrt{3}\sigma^*}\right)+1 \tag{19}$$

Let $u = \frac{\pi}{\sqrt{3}\sigma^*}$, by simplifying equation (19), we can get:

$$tu - 1 = \frac{2}{e^{tu} - 1}$$

The theorem is proved.

5. Numerical experiment

We downloaded the price data of soybean meal futures from the website of Dalian Commodity Exchange in China (www.dce.com.cn), and then we take the data into the model for analysis.

5.1 Numeral calculations

Firstly, we draw a time series figure of price data from 2001 to 2016 on a quarterly basis. And then we smoothed the uncertain time series with Equation (4). The time series $\{Y_t\}$ and $\{Z_t\}$ shows as following:

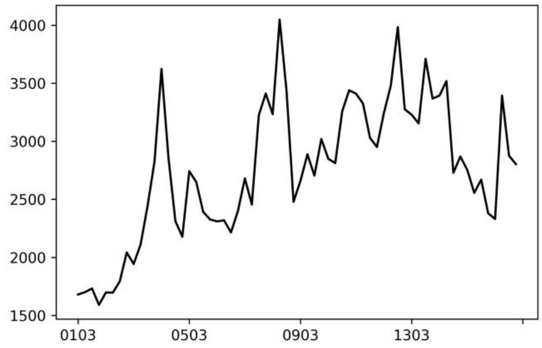

Fig. 1 Y_t

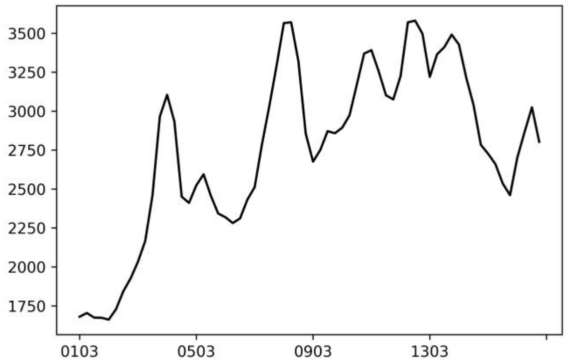

Fig. 2 Z_t

Next, we extracted the trend term T_t from the time series Z_t, Fig. 3 shows the graphics of W_t.

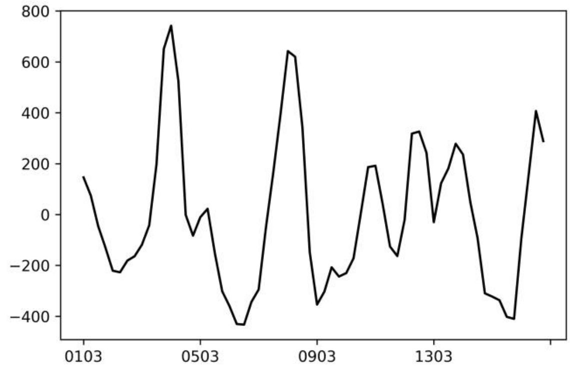

Fig. 3 $W_t = Z_t - T_t$

According to the Algorithm 1, we can get the period of W_t is $T = 17$. From this we can get the seasonal term S_t with Equation (2). Fig. 4 shows the graphics of X_t.

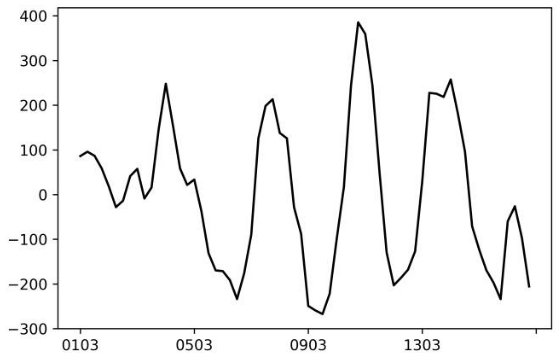

Fig. 4 $X_t = W_t - S_t$

At last, when we were fitting the uncertain term X_t, we let the order of autoregressive model (10) equal to the periodic $T = 17$. At the same time, took the solution of Equation (11), which is $\vec{c} = (c_1, c_2, \cdots, c_{17})$ as shown in Table 1 into the model, we get the prediction result of X_t, and if we consider seasonal term and trend term, we get the prediction result of Z_t.

Table 1. The parameters of model (10)

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
0.5439	0.1675	-0.3551	-0.1365	-0.1179	-0.2487	0.1867	-0.2946	-0.2860
c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	c_{17}	
-0.0880	0.0382	-0.2030	0.0647	0.2535	-0.4023	0.0146	0.0183	

The fitting results are shown in Fig. 5 and Fig. 6. We can see that the fitting result of the model is good, and the predicted curve is close to the actual data.

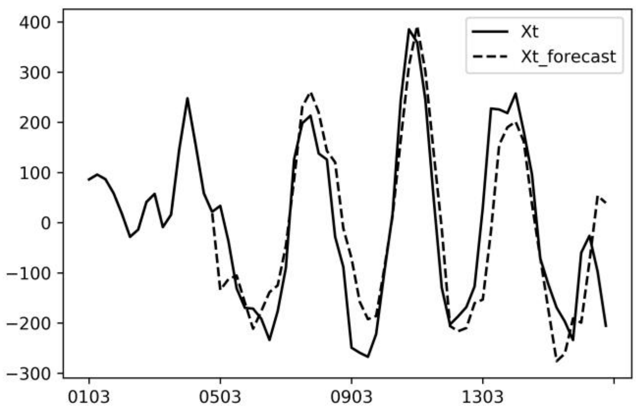

Fig. 5 X_t and its prediction result

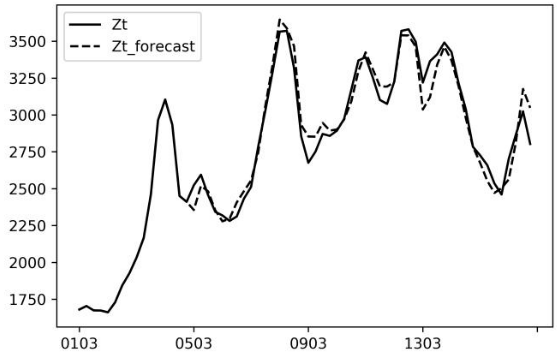

Fig. 6 Z_t and its prediction result

Taking the data of X_t , $t = 1,2,\cdots,N$, we can get the maximum likelihood estimation of the mean and variance of the uncertain term:

$$\begin{cases} e^* = 59.01 \\ \sigma^* = 383.47 \end{cases} \tag{20}$$

5.2 Forecast value and confidence interval

From the above discussion, we get that:

$$X_t = \sum_{i=1}^{n} \hat{c}_i X_{t-i} + \varepsilon_t$$

where $\hat{c}_i$ is the least squares estimator of c_i , $i = 1,2,\cdots,n$. Then the predicted value of the $(N+1)$th valueis the expectation of the $(N+1)$th value:

$$\begin{aligned} \mu_{N+1} &= E[X_{N+1}] \\ &= E\left[\sum_{i=1}^{n} \hat{c}_i X_{N+1-i} + \varepsilon_{N+1}\right] \\ &= \sum_{i=1}^{n} \hat{c}_i E[X_{N+1-i}] + E[\varepsilon_{N+1}] \\ &= \sum_{i=1}^{n} \hat{c}_i E[X_{N+1-i}] \end{aligned} \tag{21}$$

According to Equation (21), we can get:

$$\mu_{N+1} = \sum_{i=1}^{n} e^* \hat{c}_i = -82.68$$

When the seasonal term and trend term are considered, the final prediction result will be shown as follows:

$$Y_{N+1}^* = T_{N+1} + S_{N+1} + \mu_{N+1} = 2687.41$$

where T_{N+1} and S_{N+1} are the $(N+1)$th terms of season factors and trend terms, respectively.

Take α (e.g.90%) as the confidence level, and find the minimum value b such that:

$$\hat{\Phi}_{N+1}(\mu + b) - \hat{\Phi}_{N+1}(\mu - b) \geq \alpha$$

where

$$\hat{\Phi}_{N+1}(x)=\left(1+\exp\left(\frac{\pi|e-x|}{\sqrt{3}\sigma}\right)\right)^{-1}$$

is the uncertainty distribution of uncertain normal variable X_{N+1}, and its inverse uncertain distribution function is $\hat{\Phi}_{N+1}^{-1}(\alpha)$.

According to the symmetry of the normal uncertain distribution function, we can get:

$$\hat{\Phi}_{N+1}^{-1}\left(\frac{1}{2}+\frac{\alpha}{2}\right)-\hat{\Phi}_{N+1}^{-1}\left(\frac{1}{2}-\frac{\alpha}{2}\right)=(\mu+b)-(\mu-b)$$

i.e.

$$b=\frac{1}{2}\left(\hat{\Phi}_{N+1}^{-1}\left(\frac{1}{2}+\frac{\alpha}{2}\right)-\hat{\Phi}_{N+1}^{-1}\left(\frac{1}{2}-\frac{\alpha}{2}\right)\right) \quad (22)$$

where $\hat{\Phi}_{N+1}^{-1}(\alpha)$ is the inverse normal uncertain distribution function. And

$$\hat{\Phi}_{N+1}^{-1}(\alpha)=e^{*}+\frac{\sqrt{3}\sigma^{*}}{\pi}\ln\left(\frac{\alpha}{1-\alpha}\right) \quad (23)$$

According to Equation (22) and (23), we can get $b=268.61$. Hence, the 90% confidence interval of the predictive value X_{N+1} is $(-351.29, 185.93)$.

Taking the result into Equation (9), we get a 90% confidence interval of the predictive value YN+1:

$$(2418.79, 2956.02) \quad (24)$$

The final prediction result $Y_{N+1}^{*}=2687.41$, and in fact, $Y_{N+1}=2687.41$, which meets the criteria of confidence interval. It means that the model successfully predicted the real price with 90% confidence, and the prediction result is very close to the actual data.

6. Conclusion

In this paper, we study the price change of soybean meal futures from 2001 to 2016, and establish the corresponding cycle model and ARMA model to fit the historical data. From this we get the rule of price change. Soybean meal prices, for example, change over an around four-year period. In the overall trend, soybean meal prices in the early years showed an upward trend, but in recent years showed a slight decline in the trend. In addition, we use ARMA model to fit the residual term in the periodic model. Finally, we forecast the future price and give the confidence interval. The real data is in the confidence interval. The reliability of the model is illustrated.

7. Acknowledgements

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